

MATH 165

Exam 1

Fall 2019

Student name: _____

Recitation instructor: _____

Section: _____

*This exam is closed book and closed notes. No electronic devices, including calculators and headphones, are allowed. Drawing pictures to help understand and solve the questions is encouraged. Answer each question completely using exact values; you do not need to simplify your answers unless otherwise indicated. Show your work neatly, including correct notation and showing the steps in your work, as well as writing legibly; **answers without work and/or justifications will not receive credit.** Circle your final answer for each problem. The test is out of 60 points with each problem worth 10 points; the lowest scoring problem will be dropped.*

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DO NOT BEGIN THIS EXAM UNTIL INSTRUCTED TO START

Do not write
in these boxes
on the exam



score

1. Let $f(x) = Ax^2 + Bx + C$. Find constants A , B , and C so the following is true:

- $f(0) = 10$;
- the average rate of change from $x = 0$ to $x = 5$ is -2 ; and
- the average rate of change from $x = -2$ to $x = 5$ is 0 .

Write the final answer for A , B , and C in the following box:

$A =$ $B =$ $C =$

score

2. Let $f(x) = 1 + \sqrt{x}$. Use a **limit definition of the derivative** to find $f'(x)$.
(No credit will be given for finding the derivative using any other method.)

score

3. The regional manager of a small paper company is giving a guest lecture at a business class, and begins using miniature drones (nicknamed “Scott’s bots”) to throw candy bars into the audience. The drones follow a strange trajectory with the height of a drone given by $h(t) = -3t^3 - \frac{3}{2}t^2 + 6t + e^3$, where $h(t)$ is measured in *yards* and $t \geq 0$ is measured in *seconds*. Determine the *acceleration* of a drone at the time when the *velocity* is zero. Include units in your final answer.

score

4. (a) Write down the *two* different limit definitions to find the derivative of a function $f(x)$.

$$f'(x) = \lim_{\substack{\longrightarrow 0 \\ \text{---}}} \text{---}$$

$$f'(x) = \lim_{\substack{\longrightarrow x \\ \text{---}}} \text{---}$$

(b) Find $f'(x)$ for $f(x) = x^2 \sin(x^2)$. (You can use any methods/rules for derivatives such as product rule, sum rule, chain rule, quotient rule,)

(c) Find $\lim_{t \rightarrow 2} \frac{t^2 \sin(t^2) - 4 \sin(4)}{t - 2}$.

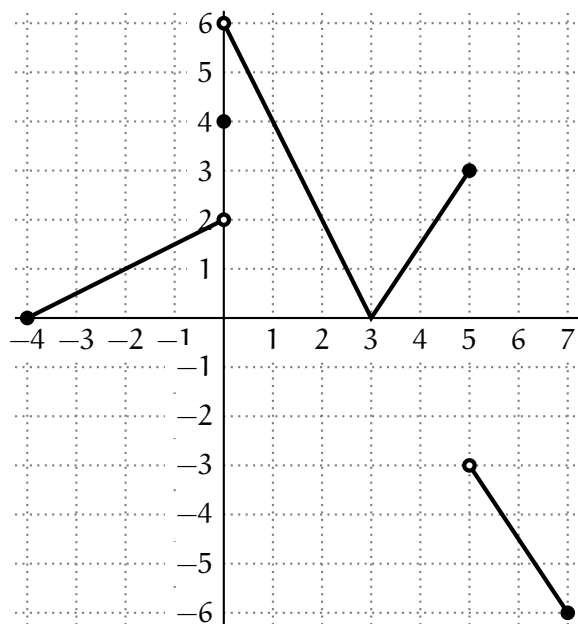
You cannot use L'Hôpital to find this limit. (Hint: Do not evaluate the limit directly. Instead, compare it to parts (a) and (b) to determine what it must equal.)

score

5. The line $y = 2x + 3$ is tangent to the graph of $y = f(x)$ at the point with $x = 4$. The line $y = 7x + 25$ is tangent to the graph of $y = g(x)$ at the point with $x = -3$. Find an equation for the line tangent to $y = f(g(x)) + 2x$ at the point with $x = -3$.

score

6. The graph of $y = P(x)$ for the interval $-4 \leq x \leq 7$ is shown below.



Use the graph to answer the following.

(a) Evaluate $\lim_{x \rightarrow 3^-} P(P(x))$, or explain why the limit does not exist.

(b) Let $Q(x) = \frac{1}{P(x)}$. Calculate $Q'(1)$ or explain why the derivative does not exist.

(c) Find, if possible, $\frac{d}{dx}(|P(x)|)$ at $x = 5$. Briefly explain your answer.

score

7. Find values of a, b, c which are *positive* and so that the following function is continuous.

$$f(x) = \begin{cases} \frac{1 - \cos(ax)}{x^2} & \text{if } x < 0; \\ 8 & \text{if } 0 \leq x \leq 1; \\ \frac{(x - b)(x + c)}{x^2 + 3x - 4} & \text{if } 1 < x. \end{cases}$$

Justify your answers by using limits. You can not use L'Hôpital to evaluate any limit.

Write the final answer for a, b , and c in the following box:

$a =$ $b =$ $c =$

score