

MATH 165
Exam 1
Spring 2024

Student name: _____
Recitation instructor: _____
Section: _____

*This exam is closed book and closed notes. No electronic devices, including calculators and headphones, are allowed. Drawing pictures to help understand and solve the questions is encouraged. Answer each question completely using exact values; you do not need to simplify your answers unless otherwise indicated. Show your work neatly. Include correct notation. Show the steps in your work. Write legibly. **Answers without work and/or justifications will not receive credit.** Circle your final answer for each problem. Each problem is worth 10 points.*

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DO NOT BEGIN THIS EXAM UNTIL INSTRUCTED TO START

Do not write
in these boxes →
on the exam

score

1. Consider the graph

$$y = x^2 - 2x - 8.$$

Let L_1 be the secant line to this graph through the points where $x = a$ and $x = 2$. Let L_2 be the tangent line to this graph at $x = 2$. Find the value of a so the lines L_1 and L_2 are perpendicular.

score

2. Compute the following limits (without using L'Hôpital's rule).

(a) (5 points) $\lim_{x \rightarrow 0} \left(\frac{\cos(5x)}{\cos(5x) - \sin(5x)} \right)$

(b) (5 points) $\lim_{x \rightarrow \infty} \left(\frac{3x^2 + 7x - 3}{4x^2 + 2x + 1} \right)$

score

3. Consider the function $f(x)$ given by

$$f(x) = \sqrt{2x + 1}.$$

Using the limit definition of the derivative, find $f'(4)$ and $f'(12)$.

Note: No credit will be given for derivatives not computed using the limit definition!

score

4. Find the equation for the tangent line to the graph of the derivative of $f(x) = e^{4x} - e^{2x} + e^x - e^{-3x}$ at $x = 0$.

score

5. Let the function $f(x)$ be defined by:

$$f(x) = \begin{cases} (1 + e^{5x}) \cos x & \text{if } x \leq 0 \\ \frac{2 - Ax}{3x^2 + x + 1} & \text{if } x > 0 \end{cases}$$

(a) (4 points) Is $f(x)$ continuous at $x = 0$? Use limits to support your answer.

(b) (6 points) Find the value of A that makes $f(x)$ differentiable at $x = 0$. Use limits to support your answer.

score

6. A gymnast throws a ball vertically into the air and catches it. The height of the ball (in meters) above the ground is given by

$$s(t) = -t^2 + 4t + \frac{3}{2},$$

where the time t is measured in seconds. Answer the following questions.

All final answers should be integers or fully reduced fractions and include units.

(a) (5 points) What is the maximum height above the ground that the ball reaches?

(b) (5 points) What is the velocity of the ball when it is caught by the gymnast $\frac{3}{2}$ meters above the ground?

score
