MATH 165 Student name: _____

Exam 1 Recitation instructor: _____

Fall 2021 Section: _____

This exam is closed book and closed notes. No electronic devices, including calculators and headphones, are allowed. Drawing pictures to help understand and solve the questions is encouraged. Answer each question completely using exact values. You do not need to simplify your answers unless otherwise indicated. Show your work neatly. Include correct notation, and show the steps in your work. Write legibly. **Answers without work and/or justifications will not receive credit**. *Circle your final answer for each problem. Each problem is worth* 10 *points.*

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DO NOT BEGIN THIS EXAM UNTIL INSTRUCTED TO START

Do not write in these boxes on the exam ⟶

score

1. Let L_1 be the secant line to the graph of $y = x^2 - 6x + 12$ through the two distinct points where $x = 2$ and $x = a$.

Let L_2 be the tangent line to the graph at $x = 2$.

Find the value of the constant a so that L_1 and L_2 are perpendicular.

score

2. Consider the function

$$f(x) = \begin{cases} 3 - e^{-x} & \text{if } x < 0 \\ \frac{1-\cos(2x)}{x^2} & \text{if } x > 0 \end{cases}$$

(a) Find $\lim_{x \to 0} f(x)$, or explain why the limit does not exist. No credit will be given for the use of L'Hopital's Rule.

(b) Find the horizontal asymptote(s) of $f(x)$, if any exist. Use limits to justify your answer.

score

3. (a) For $P(t) = (4t^2 + 3)(2t^3 - t)^{12}$, find $P'(t)$. After taking the derivative(s), do not simplify your answer.

(b) Find $\dfrac{d^2}{dx^2}\left(e^{4x} - \tan(3x) - 8\sqrt{x} + \dfrac{\pi}{4}\right)$. After taking the derivative(s), do not simplify your final answer.

score

4. Find the derivative of the function $f(x) = \sqrt{3x}$ using the (limit) definition of the derivative. No credit will be given for any other method.

score

5. Consider the function $f(x)$.

$$f(x) = \begin{cases} 2x^2 + 3x + 1 & \text{if } x < -1 \\ |x| & \text{if } -1 \leqslant x \leqslant 1 \\ \sin(x-1) + 1 & \text{if } 1 < x \end{cases}$$

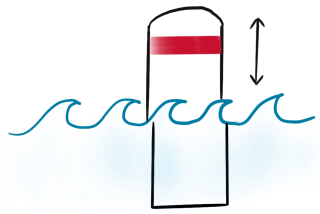
(a) Check if $f(x)$ has a derivative at $x = -1$. If $f(x)$ has a derivative at $x = -1$, then give the derivative. If not, explain why.

(b) Check if $f(x)$ has a derivative at $x = 1$. If yes, then give the derivative of $f(x)$ at $x = 1$. If not, explain why.

(c) Does $f(x)$ have a derivative at all remaining values of x? Explain.

score

6. A buoy in the ocean bobs ups and down as the waves hit the buoy. The height of the buoy is given by



$$h(t) = 3e^{-t}\sin(t)$$

where t is measured in seconds and $h(t)$ is measured in centimeters.

(a) Find the instantaneous rate of change of the height of the buoy with respect to time at $t = \pi/3$ seconds. Include units with your answer.

(b) Is the buoy rising or falling at $t = \pi/3$ seconds? How do you know?

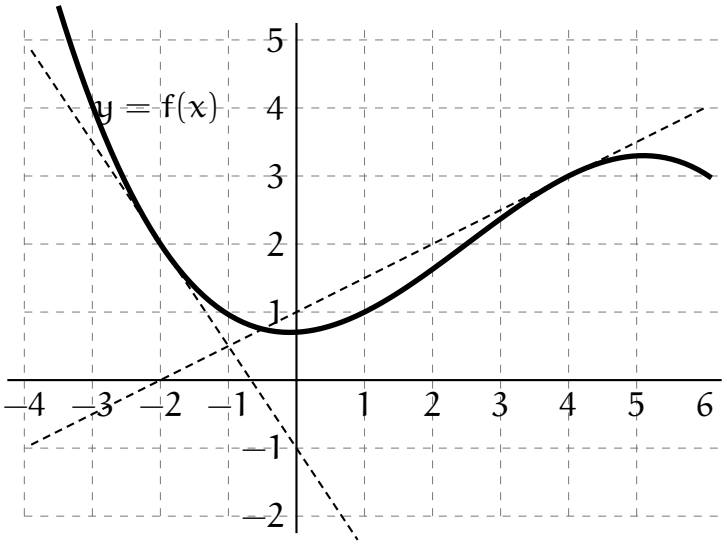
(c) For what value(s) of t, $0 \leqslant t \leqslant \pi$, is the acceleration of the buoy equal to 0 cm/sec^2?

score

7. Let the $f(x)$ be the graph of the curve shown here. The graphs of the tangent lines to $y = f(x)$ at $x = -2$ and $x = 4$ are also shown. Let $g(x)$ be a differentiable function ***not shown on the graph***.

- The tangent line to $y = g(x)$ at $x = -2$ is $y = 4x + 5$.
- The tangent line to $y = g(x)$ and $x = 4$ is $y = -5x + 26$.

If $R(x) = \dfrac{f(x)}{g(x)}$, write the equation of the line tangent to $y = R(x)$ at $x = 4$.



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