

MATH 165

Exam 2

Fall 2022

Student name: _____

Recitation instructor: _____

Section: _____

*This exam is closed book and closed notes. No electronic devices, including calculators and headphones, are allowed. Drawing pictures to help understand and solve the questions is encouraged. Answer each question completely using exact values; you do not need to simplify your answers unless otherwise indicated. Show your work neatly. Include correct notation. Show the steps in your work. Write legibly. **Answers without work and/or justifications will not receive credit.** Circle your final answer for each problem. Each problem is worth 10 points.*

1	
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DO NOT BEGIN THIS EXAM UNTIL INSTRUCTED TO START

Do not write
in these boxes →
on the exam

score

1. Find the linearization of $f(x) = (x^3 + 1)^{1/2}$ at $x = 2$ and use this to estimate $f(2.05)$.

score

2. Let $xy - y^2 = 8$.

a) Find $\frac{dy}{dx}$.

b) Find an equation for the tangent line at the point $(6,4)$.

score

3. Use **logarithmic differentiation** to compute the derivative of

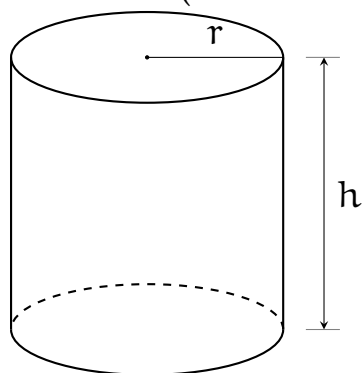
$$f(x) = \frac{x^4(x-3)^{\arctan(2x)}}{\sqrt{1+x^2}}.$$

No credit will be given for other differentiation methods. Your answer should be a function of x alone. **Do not simplify your answer.**

score

4. An object in the shape of a cylinder has a volume of $72\pi \text{ m}^3$. Its radius and height are changing in such a way that this volume remains constant.

If the height is increasing at a steady rate of 5 m/min , find the rate of change of the radius at the moment the height is 8 meters . Include units in your answer. (Volume of a cylinder is: $V = \pi r^2 h$)



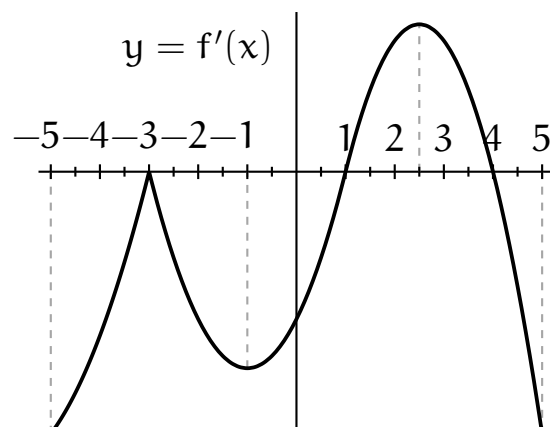
score

5. Find the absolute maximum and minimum values of the function

$$f(x) = \frac{-x^2 + 4x + 1}{x + 1} \text{ on the interval } [0, 4].$$

score

6. Let $f(x)$ be a continuous, differentiable function for $-5 \leq x \leq 5$. Consider the graph of $y = f'(x)$, **the derivative** of $f(x)$ as shown. Use the graph to answer the following questions.



- (a) (4 points) For each pair of values, **circle the** $<$, $=$, **or** $>$ to give the relationship that exists between the values, **or circle "more information needed."**

$f(-5)$	$<$	$=$	$>$	$f(-3)$	more information needed
$f(-2)$	$<$	$=$	$>$	$f(0)$	more information needed
$f(1)$	$<$	$=$	$>$	$f(4)$	more information needed
$f(4)$	$<$	$=$	$>$	$f(5)$	more information needed

- (b) (2 points) Identify the open intervals for which $f(x)$ is concave up.

- (c) (2 points) Identify the open intervals for which $f(x)$ is concave down.

- (d) (2 points) Identify the x -values for the inflection points of $f(x)$.

score

7. Determine the point(s) on the curve $y = x^2 - \frac{1}{2}$ that are closest to the point $(0, 4)$. Justify your answer using appropriate derivative/limit computations.

Note: The distance between (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

(Hint: Minimize the square of the distance.)

score
