MATH 165
Exam 1
Spring 2019

Student name: _____
Recitation instructor: _____
Section: _____

This exam is closed book and closed notes. No electronic devices, including calculators, are allowed. Drawing pictures to help understand and solve the questions is encouraged. Answer each question completely using exact values; you do not need to simplify your answers unless otherwise indicated. Show your work neatly, including good notation showing the steps in your work, as well as writing legibly; **answers without work and/or justifications will not receive credit**. *Circle your final answer for each problem. Each problem is worth 10 points, for a total of 60 points.*

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DO NOT BEGIN THIS EXAM UNTIL INSTRUCTED TO START

Do not write in these boxes on the exam →

score

1. Using the ***limit definition*** of the derivative, find the derivative of $f(x) = 2x^2 - 3x + 1$. (No credit will be given for the derivative found by any other method.)

score

2. A particle's position (in meters) at time t (seconds) is given by the function

$$s(t) = \frac{4}{t-2} + a\sin t + \tfrac{1}{3}be^{3t}, \qquad \text{for } -1 < t < 1,$$

where a and b are constants. Find a and b so that at time $t = 0$ the velocity is $0\ \frac{m}{sec}$ and acceleration is $0\ \frac{m}{sec^2}$.

score

3. Let $f(x) = \big(g(x)\big)^2$ where the function $g(x)$ satisfies the following:

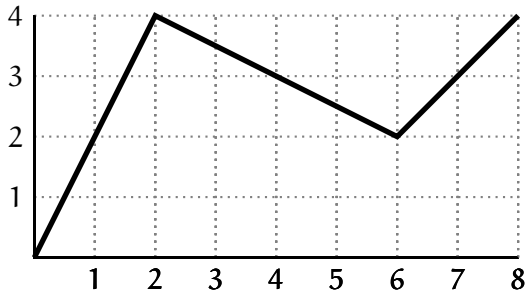
- $g(A) + g(B) = 4$,
- the average rate of change of $g(x)$ from $x = A$ to $x = B$ is 6, and
- the instantaneous rate of change of $g(x)$ at $x = B$ is 3.

(a) Find the average rate of change of $f(x)$ from $x = A$ to $x = B$. Simplify your answer as much as possible using the information given above.

(b) Given the instantaneous rate of change of $f(x)$ at $x = B$ matches the average rate of change of $f(x)$ from $x = A$ to $x = B$ (the answer from part (a)), determine $g(B)$.

score

4. The function $f(x)$ is defined for $0 \leqslant x \leqslant 8$ and is shown in the graph below. Use the graph to answer the following questions.



(a) List all values of x, with $0 \leqslant x \leqslant 8$, for which $g(x) = \dfrac{1}{f(x) - 2}$ is undefined.

(b) For *each* answer from part (a), determine whether the limit of $g(x)$ as we approach that value is ∞, $-\infty$, or undefined. Briefly justify your answer.

(c) Let $h(x) = f\big(2x^3 f(x)\big)$, determine $h'(1)$.

score

5. The line $y = 3x + 1$ is tangent to the curve $y = f(x)$ at $x = 2$. The line $x + y = 5$ is tangent to the curve $y = g(x)$ at $x = 3$. Find the tangent line to the curve $y = f(g(x))$ at $x = 3$.

score

6. This problem concerns the following piece-wise function.

$$f(x) = \begin{cases} x+3 & x < -3 \\ -10 & x = -3 \\ \dfrac{(x+2)^2 + x - 4}{x - 2x^2}, & -3 < x < 0 \\ 2 & x = 0 \\ \dfrac{\sin(5x)}{\tan(x)\cos(x)}, & 0 < x \leqslant \pi/4 \end{cases}$$

Without using L'Hôpital's rule, answer the following questions.

(a) Determine whether $f(x)$ is continuous, has a jump discontinuity, or a removable discontinuity at $x = -3$. (Justify your answer using limits.)

(b) Determine whether $f(x)$ is continuous, has a jump discontinuity, or a removable discontinuity at $x = 0$. (Justify your answer using limits.)

(c) If the answer to (a) and/or (b) was a removable discontinuity, how should the function be defined at the point(s) to make it continuous?

score