MATH 165
Exam 1
Spring 2020

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Recitation instructor: ____________
Section: ____________

This exam is closed book and closed notes. No electronic devices, including calculators and headphones, are allowed. Drawing pictures to help understand and solve the questions is encouraged. Answer each question completely using exact values; you do not need to simplify your answers unless otherwise indicated. Show your work neatly, including correct notation and showing the steps in your work, as well as writing legibly; **answers without work and/or justifications will not receive credit.** *Circle your final answer for each problem. Each problem is worth* 10 *points.*

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DO NOT BEGIN THIS EXAM UNTIL INSTRUCTED TO START

Do not write in these boxes on the exam ⟶

score

1. Using the ***limit definition*** of the derivative, find the derivative of

$$f(t) = 2 + \sqrt{5 - 3t}.$$

(No credit will be given for the derivative found by any other method.)

$$f'(t) = \lim_{h \to 0} \frac{f(t+h) - f(t)}{h}$$

$$= \lim_{h \to 0} \frac{\left(2 + \sqrt{5-3(t+h)}\right) - \left(2 + \sqrt{5-3t}\right)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{5-3(t+h)} - \sqrt{5-3t}}{h} \cdot \frac{\sqrt{5-3(t+h)} + \sqrt{5-3t}}{\sqrt{5-3(t+h)} + \sqrt{5-3t}}$$

$$= \lim_{h \to 0} \frac{(5-3(t+h)) - (5-3t)}{h\left(\sqrt{5-3(t+h)} + \sqrt{5-3t}\right)}$$

$$= \lim_{h \to 0} \frac{\cancel{5} - \cancel{3t} - 3h - \cancel{5} + \cancel{3t}}{h\left(\sqrt{5-3(t+h)} + \sqrt{5-3t}\right)}$$

$$= \lim_{h \to 0} \frac{-3\cancel{h}}{\cancel{h}\left(\sqrt{5-3(t+h)} + \sqrt{5-3t}\right)}$$

$$= \lim_{h \to 0} \frac{-3}{\sqrt{5-3(t+h)} + \sqrt{5-3t}}$$

$$= \frac{-3}{\sqrt{5-3t} + \sqrt{5-3t}}$$

$$= \boxed{\frac{-3}{2\sqrt{5-3t}}}$$

score

2. Find all horizontal asymptotes of the graph of

$$f(x) = \frac{2x^3 - 3}{|x|^3 + 1}.$$

Justify your response using limits.

note $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x \leq 0 \end{cases}$

$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{2x^3 - 3}{x^3 + 1} \cdot \frac{1/x^3}{1/x^3}$$

$$= \lim_{x \to \infty} \frac{2 - 3/x^3}{1 + 1/x^3}$$

$$= \frac{2 - 0}{1 + 0} = 2$$

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{2x^3 - 3}{-x^3 + 1} \cdot \frac{1/x^3}{1/x^3}$$

$$= \lim_{x \to -\infty} \frac{2 - 3/x^3}{-1 + 1/x^3}$$

$$= \frac{2 - 0}{-1 + 0} = -2$$

$$\boxed{\text{Horizontal asymptotes of } 2, -2}$$

score

3. An object moves along the horizontal s-axis. Its position relative to the origin at time $t \geqslant 0$ seconds is given by

$$s(t) = (t^3 - 9t^2 + 24t + 4) \text{ meters}$$

(a) Find all t value(s) for which the velocity of the object is 0 m/sec.

$\hookrightarrow s'(t) = 0$

$$s'(t) = 3t^2 - 18t + 24$$
$$= 3(t^2 - 6t + 8)$$
$$= 3(t-2)(t-4) = 0$$

happens at times $\boxed{t = 2, 4}$

(b) Is the object moving from right to left or from left to right at time $t = 3$ seconds?

$$s'(3) = 3(3-2)(3-4) = -3$$

showing it is moving in negative direction

so going from right to left

(c) For what value(s) of $t \geqslant 0$ is the acceleration of the object equal to 0 m/sec^2?

$\hookrightarrow s''(t) = 0$

$$s''(t) = 6t - 18$$
$$= 6(t-3) = 0$$

happens at time $\boxed{t = 3}$

score

4. Given that $y = 2x + 1$ is a tangent line to the graph of $y = f(x)$ at $x = 1$, and $y = -4x + 5$ is a tangent line to $y = g(x)$ at $x = 1$, find an equation of a tangent line to $y = f(g(x))$ at $x = 1$.

$$\left.\begin{array}{l} y = 2x+1 \\ \text{at } x=1 \\ \text{for } f(x) \end{array}\right\} \to \begin{array}{l} f(1) = 3 \\ f'(1) = 2 \end{array}$$

$$\left.\begin{array}{l} y = -4x+5 \\ \text{at } x=1 \\ \text{for } g(x) \end{array}\right\} \to \begin{array}{l} g(1) = 1 \\ g'(1) = -4 \end{array}$$

$$y = f(g(x)) \rightsquigarrow f(g(1)) = f(1) = 3$$

$$\frac{dy}{dx} = f'(g(x))g'(x) \rightsquigarrow f'(g(1))g'(1) = f'(1)g'(1) = 2(-4) = -8$$

$$y - 3 = -8(x-1)$$

or $\boxed{y = -8x + 11}$

score

5. Given that a function $f(x)$ satisfies the following:

- The average rate of change from $x = 10$ to $x = 20$ is 15, $\rightarrow \dfrac{f(20)-f(10)}{20-10} = 15$
- The average rate of change from $x = 20$ to $x = 30$ is 5, and $\rightarrow \dfrac{f(30)-f(20)}{30-20} = 5$
- The average rate of change from $x = 30$ to $x = 40$ is -2. $\rightarrow \dfrac{f(40)-f(30)}{40-30} = -2$

(a) Find the average rate of change of $f(x)$ from $x = 10$ to $x = 40$.

$$f(20)-f(10) = 150$$
$$f(30)-f(20) = 50$$
$$f(40)-f(30) = -20$$

add together $\rightarrow$ $f(40)-f(10) = 180$

$$\text{AROC from } x=10 \text{ to } x=40 = \frac{f(40)-f(10)}{40-10} = \frac{180}{30} = \frac{18}{3} = \boxed{6}$$

(b) Determine the number p so the function $g(x) = f(x) + px$ has an average rate of change of 23 between $x = 10$ and $x = 30$.

$$f(20)-f(10) = 150$$
$$f(30)-f(20) = 50$$

add together $\rightarrow$ $f(30)-f(10) = 200$

$$23 = \text{AROC of } g(x) \text{ from } x=10 \text{ to } x=30 = \frac{g(30)-g(10)}{30-10} = \frac{(f(30)+30p)-(f(10)+10p)}{20}$$

$$= \frac{f(30)-f(10)+20p}{20}$$

$$= \underbrace{\frac{f(30)-f(10)}{20}}_{200/20 = 10} + p$$

$$= 10 + p \rightarrow \boxed{p = 13}$$

score

6. Find the following derivatives.

triple product rule

(a) $\frac{d}{dx}\left(x^{2/3}e^{\pi}\left(\cos\left(\frac{x}{2}\right)-e^{-x}\right)\sin(2x)\right)$

$$\boxed{\begin{aligned} &= \frac{2}{3}x^{-1/3}e^{\pi}\left(\cos\left(\frac{x}{2}\right)-e^{-x}\right)\sin(2x) \\ &\quad + x^{2/3}e^{\pi}\left(-\frac{1}{2}\sin\left(\frac{x}{2}\right)+e^{-x}\right)\sin(2x) \\ &\quad + x^{2/3}e^{\pi}\left(\cos\left(\frac{x}{2}\right)-e^{-x}\right)\left(2\cos(2x)\right) \end{aligned}}$$

(b) $\frac{d}{d\theta}\left(\tan(3\theta)\cos(2\theta)-\sec^2(\pi\theta)+\tan\left(\frac{\theta}{7}+2\right)\right)$

$$\begin{aligned} &= 3\sec^2(3\theta)\cos(2\theta) + \tan(3\theta)\left(-2\sin(2\theta)\right) \\ &\quad - 2\sec(\pi\theta)\left(\sec(\pi\theta)\tan(\pi\theta)\right)\pi \\ &\quad + \frac{1}{7}\sec^2\left(\frac{\theta}{7}+2\right) \end{aligned}$$

$$\boxed{\begin{aligned} &= 3\sec^2(3\theta)\cos(2\theta) - 2\tan(3\theta)\sin(2\theta) \\ &\quad - 2\pi\sec^2(\pi\theta)\tan(\pi\theta) \\ &\quad + \frac{1}{7}\sec^2\left(\frac{\theta}{7}+2\right) \end{aligned}}$$

score

7. (a) List the condition(s) for a function to $f(x)$ be continuous at the point $x = c$.

$$\boxed{\lim_{x \to c} f(x) = f(c)}$$

- limit as $x \to c$ of $f(x)$ exists
- f is defined at c
- two preceding values agree

(b) For the following function $g(x)$, list all possible values of x where the function *might* be discontinuous.

$$g(x) = \begin{cases} \dfrac{2x + a + \sqrt{x+3}}{x+2} & x > -2 \\ |x+4| + b & x \leqslant -2 \end{cases}$$

$\boxed{x = -2}$ (where piecewise parts line up; note each part is continuous in its side)

(c) Find values for a and b so that the function $g(x)$ from part (b) *is* continuous.

$$\lim_{x \to -2^-} \frac{2x+a+\sqrt{x+3}}{x+2} \longrightarrow \frac{-4+a+\sqrt{1}}{0} = \frac{a-3}{0}$$

need $a = 3$ to be continuous (if $a \neq 3$ get a vertical asymptote)

$$\lim_{x \to -2^-} \frac{(2x+3)+\sqrt{x+3}}{x+2} \cdot \frac{(2x+3)-\sqrt{x+3}}{(2x+3)-\sqrt{x+3}}$$

$$= \lim_{x \to -2^-} \frac{(2x+3)^2-(x+3)}{(x+2)((2x+3)-\sqrt{x+3})}$$

$$(2x+3)^2-(x+3) = 4x^2+12x+9-x-3 = 4x^2+11x+6 = (x+2)(4x+3)$$

$$= \lim_{x \to -2^-} \frac{\cancel{(x+2)}(4x+3)}{\cancel{(x+2)}((2x+3)-\sqrt{x+3})}$$

$$= \lim_{x \to -2^-} \frac{4x+3}{(2x+3)-\sqrt{x+3}}$$

$$= \frac{-8+3}{-1-1} = \frac{5}{2}$$

$$\lim_{x \to -2^+} (|x+4|+b) = 2+b$$

to be continuous these must match

$2+b = \frac{5}{2}$ so $b = \frac{5}{2} - 2 = \frac{1}{2}$

$$\boxed{a = 3,\ b = \frac{1}{2}}$$

score