

MATH 165
Exam 2
Spring 2022

Student name: _____
Recitation instructor: _____
Section: _____

*This exam is closed book and closed notes. No electronic devices, including calculators and headphones, are allowed. Drawing pictures to help understand and solve the questions is encouraged. Answer each question completely using exact values; you do not need to simplify your answers unless otherwise indicated. Show your work neatly, including correct notation and showing the steps in your work, as well as writing legibly; **answers without work and/or justifications will not receive credit.** Circle your final answer for each problem. Each problem is worth 10 points.*

1	
2	
3	
4	
5	
6	
7	

DO NOT BEGIN THIS EXAM UNTIL INSTRUCTED TO START

Do not write
in these boxes →
on the exam

score

1. Let $f(x) = \sqrt{x}$.

a) Find the linearization of $f(x) = \sqrt{x}$ at $x = 16$.

b) Use the linearization from part (a) to estimate $\sqrt{15}$.

score

2. Let $g(x) = \arctan(x^2) - \ln(1 + x^4)$. Find the equation of the line tangent to $y = g(x)$ at $x = 1$. Simplify your answer and evaluate any term involving an inverse trigonometric function.

score

3. Use logarithmic differentiation to compute the derivative of the following function.

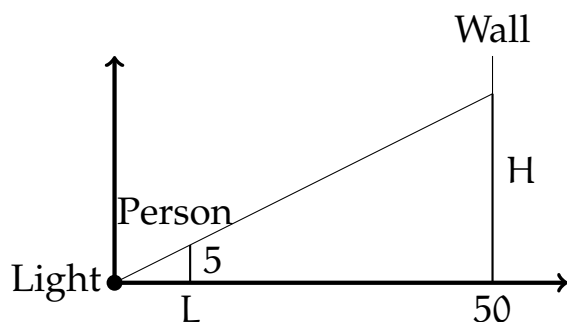
$$y = e^{-1/x^2} (\sec x)^x$$

Your final answer should be expressed completely in terms of x .

score

4. A light located at the origin shines at a wall located at $x = 50$ ft. A person 5 ft tall casts a shadow H feet high on the wall (see diagram).

If the person is currently $L = 10$ feet from the light and walking toward it at a rate of 1 ft/sec, find the rate at which the height of the shadow is changing.



score

5. Let $f(x) = (x^2 - 3)e^x$.

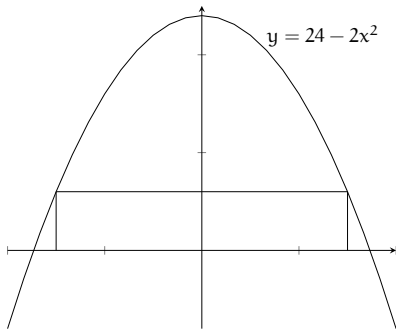
a) Find all x -values that are critical points of f .

b) Determine the open intervals on which f is monotone and state whether f is increasing or decreasing on the intervals you found.

c) Use your work in parts (a) and (b) to find all x -values corresponding to local extrema of f . State if these values correspond to a local maximum or minimum. **State the theorem you used or explain your answer.**

score

6. A rectangle has its base on the x -axis and its upper two vertices on the parabola $y = 24 - 2x^2$. What is the largest area the rectangle can have; What are its dimensions?



score

7. The first derivative of a function $f(x)$ is given by $f'(x) = (x - 2)^2(3x + 1)$.

a) Find the x -coordinates of the critical points of $f(x)$ and determine the intervals on which $f(x)$ is increasing and decreasing.

b) Find the x -coordinates of the inflection points of $f(x)$ and determine the intervals on which $f(x)$ is concave up and concave down.

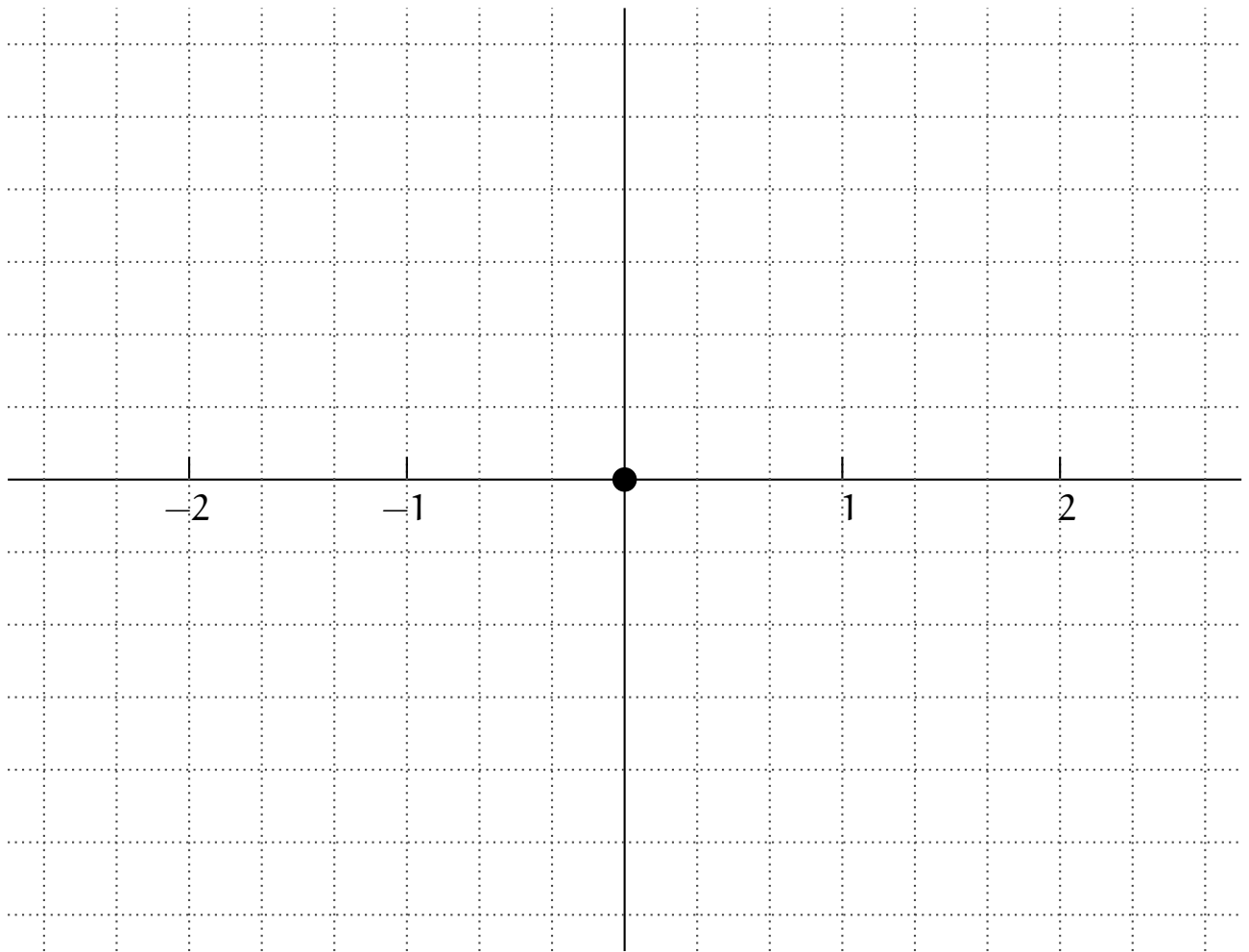
c)

(i) At critical point (enter a value)	f'' is (circle one)	Therefore, f must have (circle one)
$x =$	POSITIVE NEGATIVE	A MAXIMUM A MINIMUM

(ii) At critical point (enter a value)	we know	and by the First Derivative Test, f must have (circle one)
$x =$	$f'' = 0$	A MAXIMUM A MINIMUM NEITHER A MAX NOR A MIN

– continued on the next page –

d) Using the fact that $f(0) = 0$, sketch the general shape of the graph of $f(x)$.



score