

Differential Equations Exam

Show all work.

1. Find in implicit form the solution of the following initial value problem

$$(xe^y + 2x^2 + 6y)y' + e^y + 4xy = 0,$$

passing through the point $(2, 0)$, that is, $y = 0$ when $x = 2$.

- 2.** Find the solution to the initial value problem $xy' - 4y = 3x^6$ with $y(1) = 5$.

- 3.** Write the general solution of the differential equation,

$$y^{(5)} - 6y^{(4)} + 10y^{(3)} = 0.$$

4. A group of fish has two things affecting its population ($P(t)$). The first is internal growth which follows the logistic model ($kP(M - P)$), and the second is external fishing which reduces its population by a *fixed* amount (h) every year. So we have $P' = kP(M - P) - h$.

For these fish we have $h = 10$; when $P = 4$ then $P' = 6$; and when $P = 6$ then $P' = 8$.

- (a) Determine k and M . Rewrite (but do **not** solve) the differential equation using the values for h, k, M .

- (b) For the differential equation from (a) draw the phase diagram and find/classify all critical points.

5. Find the general solution to $x^2y' - xy - 4y^2 = 0$.

6. Find the general solution:

$$y''' + 9y' = 18x + 26e^{2x}.$$