

Differential Equations Exam

Show all work.

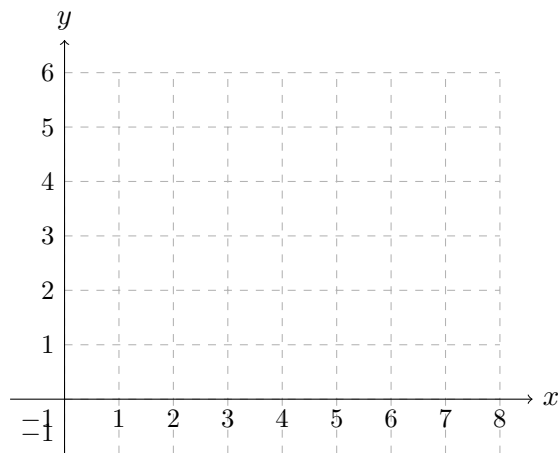
1. The following questions refer to the differential equation

$$y' = y^3 - 5y^2 + 4y.$$

- (a) Determine the fixed points of the differential equation.

- (b) Make a sketch of the phase diagram for the differential equation and indicate the stability of each of the fixed points.

- (c) Sketch the direction field for the ODE on the provided grid, highlighting any regions where slopes are zero, positive, or negative.



2. Determine the general solution to each of the following differential equations. If an initial condition is given then also determine the specific solution. Show your work. **Unjustified results may receive little or no credit.**

(a) $2x'' + 3x' - 2x = 5e^{t/2}, \quad x(0) = 0, \quad x'(0) = 1.$

(b) $x'' + 16x = \tan(4t)$.

(c) $x' = e^{2x+5t}$

- 3.** Find in implicit form the solution of the following initial value problem

$$(x^2 + 3y^2) y' + 2xy + 1 = 0,$$

passing through the point $(1, 2)$, that is, $y = 2$ when $x = 1$.

4. Find the general solution to

$$x^2y' - xy - 5y^2 = 0.$$

5. Qualitative Phase Portrait Regime Mapping: For the parametric planar system

$$\frac{d}{dt}\vec{x} = \begin{bmatrix} 0 & 1 \\ -13 & b \end{bmatrix} \vec{x},$$

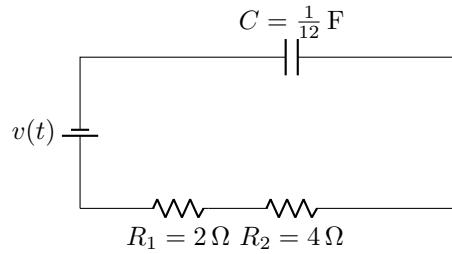
partition the b -number line into distinct topological regimes (e.g., center, stable spiral, unstable spiral, degenerate/star nodes if applicable). For each interval of b , specify:

- The trace (τ) and determinant ($\det$) signs.
- The geometric character of the origin (spiral sink/source, center, etc.).
- Sketch guidelines indicating whether trajectories spiral clockwise or counterclockwise.

6. The series circuit shown below contains a battery, a capacitor, and two resistors. At time $t = 0$ the capacitor already holds a charge of 1 C, and the battery supplies the voltage

$$v(t) = \begin{cases} 6 - 6e^{-3t} & t < 2, \\ 3 & t \geq 2. \end{cases}$$

Use the Laplace transform to find the charge $q(t)$ on the capacitor at any time $t \geq 0$.



7. Show that the Laplace transform of $f(t) = t^{3/2}$ is equal to $\frac{3\sqrt{\pi}}{4s^{5/2}}$ using the definition of the Laplace transform. (Show all of your work including each step.)